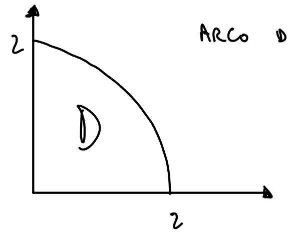
INT. DOPPI CON CAMBIAMENTO VAR.

ESEMPIO

$$\iint_D \frac{x y^2}{x^2+y^2}\, dx\, dy$$



ARCO DI CIRCONF. CENTRO $(0,0)$
$R=2$

SETTORE CIRCOLARE

- DOMINIO SODDISFA LA SIMMETRIA RADIALE

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\cos\theta \end{cases}$$

$$dx\,dy = \rho\, d\rho\, d\theta$$

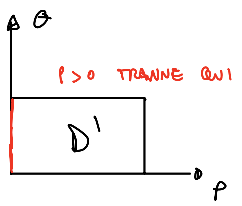
$$\rho = \sqrt{x^2+y^2}$$

$$\rho \in [0,2] \qquad \theta \in [0, \pi/2]$$

$$|J_T(\rho,\theta)| = \rho$$

$$D' = \{(\rho,\theta) : \rho \in [0,2],\ \theta \in [0,\pi/2]\}$$

$$T : D' \to D \qquad T \in C^1$$



$|J_T(\rho,\theta)| = \rho > 0$ "PER QUASI TUTTI I PUNTI DI D'"
TRANNE QUESTI

$$I = \iint_{D'} f(\rho\cos\theta, \rho\sin\theta)\, \rho\, d\rho\, d\theta$$

$$= \int_0^{\pi/2}\int_0^2 \frac{\rho\cos\theta \cdot \rho^2\sin^2\theta}{\underbrace{\rho^2\cos^2\theta + \rho^2\sin^2\theta}_{\rho^2(\cos^2\theta+\sin^2\theta) \,=\, \rho^2}}\, \rho\, d\rho\, d\theta$$

$\cos^2\theta+\sin^2\theta = 1$

$$= \left(\int_0^2 \rho^2\, d\rho\right) \cdot \left(\int_0^{\pi/2} \cos\theta\, \sin^2\theta\, d\theta\right)$$

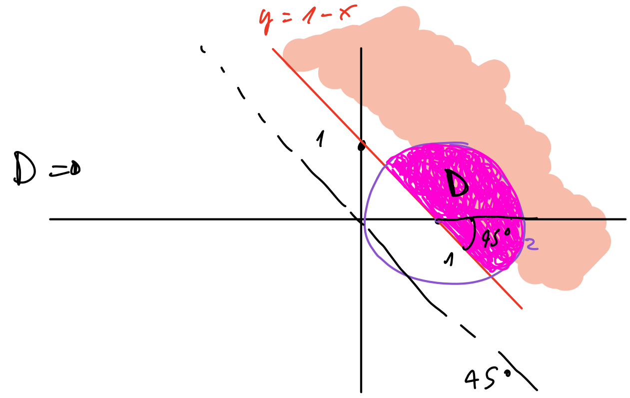
$$= \left[\frac{\rho^3}{3}\right]_0^2 \cdot \left[\frac{\sin^3\theta}{3}\right]_0^{\pi/2} = \frac{8}{3}\cdot\frac{1}{3} = \boxed{\frac{8}{9}}$$

• ESEMPI DI SIMIL - POLARI

ESEMPIO

$$\iint_D \frac{1}{(x^2+y^2-2x-1)^4}\,dx\,dy$$

$D = \{(x,y) \in \mathbb{R}^2 : x^2+y^2-2x \le 0 \ , \ x+y-1 \ge 0\}$



$x+y-1=0$

$y = 1-x$

$x^2+y^2-2x=0$

$\underbrace{x^2-2x+1}-1+y^2=0$

$(x-1)^2+y^2=1$ CIRCONF. CENTRO $(1,0)$

RAGGIO $=1$

D = 

↳ D HA UNA SIMMETRIA RADIALE? SE SÍ, RISPETTO A QUALE PUNTO? $\longrightarrow (1,0)$

$$\begin{cases} x = 1+\rho\cos\theta \\ y = \rho\sin\theta \end{cases}$$

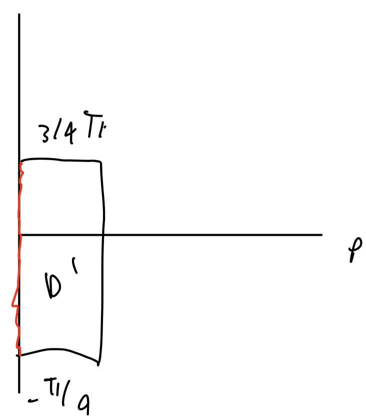
IN GENERALE

$$\begin{cases} x = x_0+\rho\cos\theta \\ y = y_0+\rho\sin\theta \end{cases}$$

$|J_T(\rho,\theta)| = \rho$

$\rho \in [0,1]$ DAL CENTRO A ORIGINE? $\theta \in \left[-\frac{\pi}{4}, \frac{3}{4}\pi\right]$

$D' = \{(\rho,\theta) : \rho \in [0,1] \ , \ \theta \left[-\pi/4, \frac{3}{4}\pi\right]\} \Longrightarrow$



$$\int_0^1\int_{-\pi/4}^{3/4\pi} \frac{1}{(\rho^2-2)^4}\cdot\rho\,d\rho\,d\theta$$

$x^2+y^2-2x-1 = (x-1)^2+y^2-1-1 = \underbrace{(x-1)^2+y^2-2}$

$\rho^2\cos^2\theta+\rho^2\sin^2\theta-2$

$=\rho^2-2$

$$= \left(\int_{-\pi/4}^{3/4\pi} d\theta\right)\left(\int_0^1 \frac{\rho}{(\rho^2-2)^4}\, d\rho\right)$$

$$= \pi \cdot \frac{1}{2}\int_0^1 \frac{2\rho}{(\rho^2-2)^4}\, d\rho$$

$g(\rho) = \rho^2 - 2$ $\quad g' = 2\rho$

$\alpha = -4$

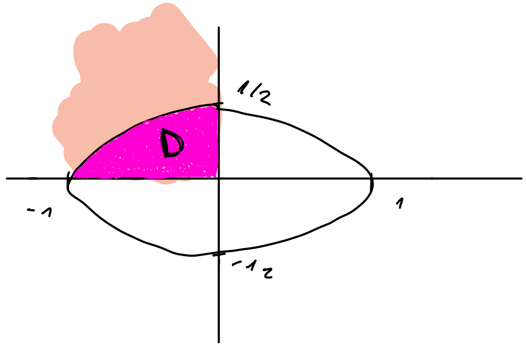
$$= \frac{\pi}{2} \frac{1}{(\rho^2-2)^3} (-3) \Big|_0^1$$

$$= -\frac{3}{2}\pi\left(-1+\frac{1}{8}\right) = \frac{3}{2}\pi\left(-\frac{1}{8}+1\right)$$

ESEMPIO

$$I = \iint_D (1+y^2)\, dx\, dy$$

$$D = \{(x,y)\in\mathbb{R}^2 : x^2+4y^2 \le 1,\ x\le 0,\ y\ge 0\}$$



$x^2+4y^2=1$

CENTRO $(0,0)$

SEMIASSI $x)$ 1

$y)$ $1/2$

• D HA SIMM. RADIALE RISPETTO A ORIGINE ? TRASFORMO IN UN CERCHIO

$$\begin{cases} x = \rho\cos\theta \\ 2y = \rho\sin\theta \Rightarrow y = \frac{1}{2}\rho\sin\theta \end{cases}$$

$x^2+4y^2=1$

$\rho^2\cos^2\theta + 4\rho^2\sin^2\theta = 1$

$\rho^2(\cos^2\theta + 4\sin^2\theta) = 1$

COSÍ NON VA.

$$\rho^2\cos^2\theta + 4\frac{\rho^2}{4}\sin^2\theta = 1$$

$\rho^2 = 1$ $\qquad 0 \le \rho \le 1$

$$J_T(\rho,\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \frac{1}{2}\sin\theta & \frac{1}{2}\rho\cos\theta \end{pmatrix}$$

$|J_T(\rho,\theta)| = \frac{1}{2}\rho$ $\qquad \theta \in \left[\frac{\pi}{2}, \pi\right]$

$$I = \int_0^1 \int_{\pi/2}^{\pi} 1 + \left(\frac{1}{2}\rho\sin\theta\right)^2 \frac{1}{2}\rho\, d\rho\, d\theta$$

$$\frac{1}{2}\int_0^1\int_{\pi/2}^{\pi}\left(1+\frac{1}{4}\rho^2\sin^2\theta\right)\rho\, d\rho\, d\theta$$

$$\frac{1}{2}\left\{\int_0^1\int_{\pi/2}^{\pi}\rho\, d\rho\, d\theta + \frac{1}{4}\int_0^1\int_{\pi/2}^{\pi}\rho^3\sin^2\theta\, d\rho\, d\theta\right\}$$

$$\frac{1}{2}\left\{\frac{1}{2}\frac{\pi}{2} + \frac{1}{4}\left(\int_0^1\rho^3\, d\rho\right)\left(\int_{\pi/2}^{\pi}\sin^2\theta\, d\theta\right)\right\}$$

$$\frac{\pi}{8} + \frac{1}{32}\int_{\pi/2}^{\pi}\sin^2\theta\, d\theta$$

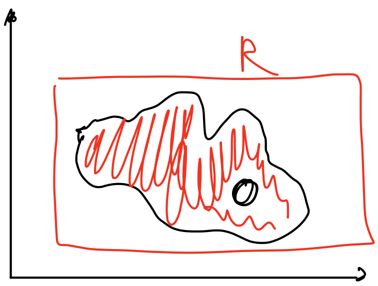
$\hookrightarrow$ PER PARTI

$$\parallel$$

$$\frac{\pi}{128} + \frac{\pi}{8}$$

$$\iint_D f(x,y)\, dx\, dy$$

D
- $\rightarrow$ RETTANGOLO
- $\rightarrow$ NORMALE
- $\rightarrow$ REGOLARE



IDEA

$$\tilde{f} : \mathbb{R} \to \mathbb{R} \qquad \tilde{f}(x,y) = \begin{cases} f(x,y), & (x,y)\in D \\ 0, & (x,y)\notin D \end{cases}$$

- DIREMO CHE f È INTEGRABILE SU D SE $\tilde{f}$ È INTEGRABILE SU RETTANGOLO R

$\Omega \subset \mathbb{R}^2$ CON Ω LIMITATO.

$\hookrightarrow$ QUANDO Ω È MISURABILE? QUANDO RIESCO A CALCOLARE L'AREA DI Ω?

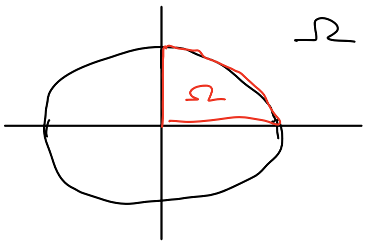
$\hookrightarrow$ SE Ω È REGOLARE $\Rightarrow$ AREA$(\Omega) = \iint_\Omega 1\, dx\, dy$

$$\boxed{f(x,y)=1}$$

- Ω È MISURABILE QUANDO $f=1$ È INTEGRABILE SU Ω

$$\text{AREA}(\Omega) = \iint_\Omega 1\, dx\, dy$$

ESEMPIO SEMI ASSI $a = x$ $b = y$



$$\text{AREA}(\Omega) = \iint_{\Omega} 1 \, dx\,dy$$

• SIMIL POLARE

$$\begin{cases} x = a \rho \cos\theta \\ y = b \rho \sin\theta \end{cases}$$

$$J_T(\rho, \theta) = \begin{pmatrix} a\cos\theta & -a\rho\sin\theta \\ b\sin\theta & b\rho\cos\theta \end{pmatrix}$$

$$|J_T(\rho,\theta)| = a \cdot b \cdot \rho$$

$$I = \int_0^1 \int_0^{\pi/2} 1 \cdot a \cdot b \cdot \rho \, d\rho\, d\theta = ab \underbrace{\left(\int_0^1 \rho \, d\rho \right)}_{= 1/2} \underbrace{\left(\int_0^{\pi/2} d\theta \right)}_{= \pi/2}$$

$$= \frac{\pi a b}{4}$$

AREA ELLISSE: $\pi \cdot a \cdot b$

BARICENTRO DI UNA FIGURA
↓
SOTTOINSIEME LIMITATO DEL PIANO

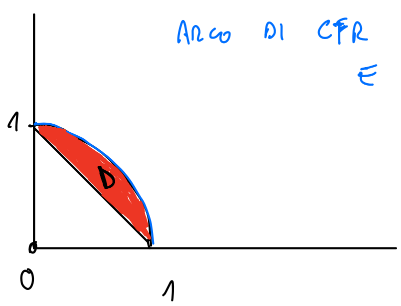
$D \subset \mathbb{R}^2$ D LIMITATO MISURABILE

• IL BARICENTRO DI D É (x_0, y_0)

$$x_0 = \frac{\iint_D x \, dx\,dy}{\text{AREA}(D)} \qquad y_0 = \frac{\iint_D y \, dx\,dy}{\text{AREA}(D)}$$

ESEMPIO

ARCO DI CFR DI CENTRO $(0,0)$
E RAGGIO $= 1$



• TROVARE BARICENTRO DI D